

Influence of the re-scattering process on polarization observables in reaction $\gamma d \rightarrow pp\pi^-$ in Δ -resonance region.¹

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Abstract

The influence of the pion-nucleon and nucleon-nucleon re-scattering effects on the polarization observables of the reaction $\gamma d \rightarrow pp\pi^-$ in Δ – isobar region is investigated. Pion-nucleon and nucleon-nucleon re-scattering are studied in the diagrammatic approach. Relativistic-invariant forms of the photoproduction and pion-nucleon scattering operators are used. The unitarization procedure in K -matrix approach is applied for the resonance partial amplitudes. It is shown a considerable influence of the re-scattering on the polarization observables of this reaction in Δ – resonance region for large momenta of the final protons.

1

Recently the experiment on studying the reaction $e\vec{d} \rightarrow e'pp\pi^-$ at photon point on the internal polarized deuterium target at the VEPP-3 storage ring has been performed [1, 2]. In the experiment two protons with momenta exceeding $300 \text{ MeV}/c$ were detected in coincidence. The obtained differential cross sections and asymmetry components, especially a_{20} , differ considerably from those calculated in the spectator model. In such a situation it seems to be reasonable to take into account the Final State Interaction (FSI) of the reaction, namely pion-nucleon and nucleon-nucleon re-scattering. In the work [3] the spin-averaged differential cross-section of the π^- -photoproduction reaction on the deuteron

$$\gamma d \rightarrow pp\pi^- . \quad (1)$$

was calculated in diagrammatic approach with an account of FSI. The contribution of FSI to the polarization observables in deuteron electro- and photo-disintegration reactions

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are calculated in Refs. [4, 8]. In present work an influence of FSI on the polarization observables of the reaction (1) is studied. The results of our calculations can be used as a basis for the simulation of the behavior of polarization observables for specified phase-space elements of the reaction (1), for comparison with existent experimental data and for an optimal planning of new polarization experiments with large momenta of detected protons. Such a comparison can help to determine whether it is necessary to include into consideration more complicated mechanisms of the reaction (1).

A description of the used model and the expressions for the amplitudes of various re-scattering mechanisms are given in section 2. Section 3 is devoted to the determination of the helicity amplitudes and polarization observables. The results of the calculations are presented in section 4.

2

To derive an amplitude for the reaction (1) we will use a diagrammatic approach developed in the works [9, 10]. The diagrammatic approach in the analysis of the π -meson photo-production reaction on the deuteron was applied in Refs. [3, 11]. We will take into consideration the contribution of the diagrams shown in Fig.1.

The first diagram corresponds to the spectator model. This model sufficiently well describes experimental data for kinematics close to the quasi-free π -meson photoproduction. In the case of sufficiently large momenta of both nucleons (above 200 MeV/c) one must take into account contributions of next diagrams which describe final state re-scattering.

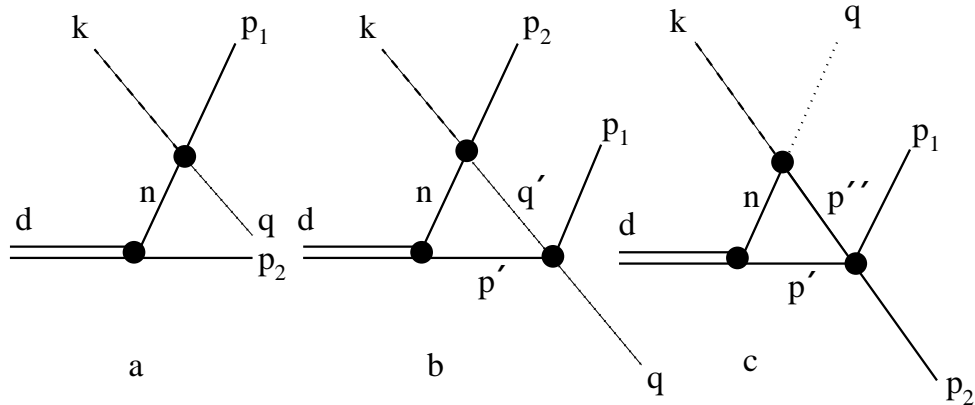


Fig.1

Qualitatively explaining, this is because a probability to find a nucleon-spectator in the deuteron reduces with an increase of its momentum. In the spectator model this leads

to a reduction of the reaction amplitude. In the same time the re-scattering amplitude is determined by an integral over nucleon momentum in the deuteron. The main contribution to this integral even for large nucleon momenta in the final state may come from a low-momentum part of the deuteron wave function. That is why the role of re-scattering effects increases with nucleons momenta.

A non-relativistic form of the photoproduction amplitude relevant in the Δ -isobar region in an arbitrary frame for small p^2/m^2 , where p and m being nucleon momentum and nucleon mass, was used in [3, 11] to construct analytical expressions corresponding to the diagrams depicted in Fig.1. For a description of the resonance partial amplitude $M_{1+}^{\frac{3}{2}}$ the Δ -mass was taken with an imaginary component, which is a function of isobar energy. The used parametrization of this function did not allow to describe the photoproduction of both charged and neutral π -mesons in a unified way. We used the amplitude of π -meson on nucleon photoproduction from [12]. This amplitude is written in relativistic-invariant and gauge-invariant form, it realizes a pseudovector variant of πN -interaction and takes into account a contribution of Born diagrams in s -, t - and u - channels, as well as contributions of s - and u - channels of Δ -isobar and of t - channel exchange of ω - and ρ - mesons. For the resonance partial amplitudes $E_{1+}^{\frac{3}{2}}$ and $M_{1+}^{\frac{3}{2}}$ a procedure of unitarization in K -matrix approach was applied. This allowed to avoid an introduction of the additional imaginary component to the Δ -isobar mass and permitted to describe the process of photoproduction of charged and neutral π -mesons in a unified way.

Pion-nucleon scattering was described by a relativistic-invariant amplitude, given in [13], where also a pseudovector variant of πN interaction was realized. In that paper the contributions of Born parts in s -, t -, u - channels, Δ -isobar in s - and u - channels, t -channel exchange of σ - and ρ - mesons were accounted for. For the resonance partial-wave amplitude P_{33} a unitarization procedure in a K -matrix approach was used in a similar way as in the case of the photoproduction of π -mesons on nucleons.

Nucleon-nucleon scattering amplitude was presented in a form of multipole expansion with partial waves of upto $L = 2$ orbital momentum. Partial phase shifts of nucleon scattering were taken from [14].

The wave function of the final NN -state in a coupled basis in spin and isospin spaces which satisfies the symmetry rules with respect to a permutation of identical nucleons has the form:

$$|p_1, p_2, sm_s, tm_t\rangle = \frac{1}{\sqrt{2}} \left(|p_1\rangle^1 |p_2\rangle^2 + (-)^{1+s+t} |p_2\rangle^1 |p_1\rangle^2 \right) |sm_s\rangle |tm_t\rangle, \quad (2)$$

where p_1, p_2 - nucleons momenta, s, m_s - spin of nucleon pair and its projection on z -axis, t, m_t - isospin of nucleon pair and its projection on z -axis. In this case the amplitude of the reaction (1) in the spectator model in a coordinate system where z -axis is along the direction of γ -quantum and d is deuteron momentum has the form [15]:

$$T^{spect}(p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, m_d) =$$

$$\begin{aligned}
&= - \sum_{m'_s} \langle sm_s | \left[\sqrt{\frac{E_{p_2}}{E_{d-p_2}}} T_{\gamma n \rightarrow p \pi^-}^1 (p_1, q; d - p_2, k, \lambda_\gamma) \Psi_{m'_s, m_d} \left(\frac{1}{2} (d - 2p_2) \right) + \right. \\
&\quad \left. + (-)^s \sqrt{\frac{E_{p_1}}{E_{d-p_1}}} T_{\gamma n \rightarrow p \pi^-}^1 (p_2, q; d - p_1, k, \lambda_\gamma) \Psi_{m'_s, m_d} \left(\frac{1}{2} (d - 2p_1) \right) \right] |1m'_s\rangle . \quad (3)
\end{aligned}$$

Here p_1 , p_2 and q are the momenta of final nucleons and π -meson, k , λ_γ – momentum and helicity of γ -quantum, d , m_d – deuteron momentum and z -projection of deuteron spin, s , m_s – spin of the nucleon pair in final state and its projection on z -axis, m'_s – projection on z -axis of the total spin of nucleons in deuteron, E_p – “on-shell” energy of a nucleon having a momentum p . $T_{\gamma n \rightarrow p \pi^-}^1 (p_1, q; d - p_2, k, \lambda_\gamma)$ – π^- -meson on neutron photoproduction amplitude, which is considered as an operator acting on spin variables of a first nucleon in a two-nucleon system. The terms $\Psi_{m'_s, m_d} (p)$ are deuteron formfactors in a coupled-basis presentation. They can be expressed through the S - and D - deuteron wave-functions as follows [15]:

$$\Psi_{m'_s, m_d} (p) = (2\pi)^{\frac{3}{2}} \sqrt{2E_D} \sum_{L=0,2} \sum_{m_L} i^L u_L (p) Y_{L m_L} (\hat{p}) \langle L m_L 1 m'_s | 1 m_d \rangle , \quad (4)$$

where E_D – deuteron energy, \hat{p} – a unitary vector in the direction of the momentum p . In present work the deuteron wave functions of Bonn potential (full model) [16] were used in calculations. Pion-nucleon re-scattering is described by the diagram in Fig. 1b and the one with identical nucleons in final state been permuted. For the reaction (1) there is a channel of pion-nucleon re-scattering without charge-exchange: $\pi^- p \rightarrow \pi^- p$, and the one with charge-exchange: $\pi^0 n \rightarrow \pi^- p$. Taking this into account and using the identity of final nucleons one can express the contribution of pion-nucleon re-scattering into the amplitude of the reaction (1) as follows:

$$\begin{aligned}
&T^{\pi N} (p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, m_d) = \\
&\int \frac{d^4 p'}{(2\pi)^4} \sum_{m'_s} \frac{\langle sm_s | [T_{\pi N}^1 (p_1, q; p', q') T_{\gamma N}^2 (p_2, q'; d - p', k, \lambda_\gamma)] |1m'_s\rangle}{(n^0 - E_{d-p'} + i\epsilon) (p'^0 - E_{p'} + i\epsilon) (q'^2 - m_\pi^2 + i\epsilon)} V (d - p', p'; m'_s, m_d) + \quad (5) \\
&+ (-)^s \int \frac{d^4 p'}{(2\pi)^4} \sum_{m'_s} \frac{\langle sm_s | [T_{\pi N}^1 (p_2, q; p', q') T_{\gamma N}^2 (p_1, q'; d - p', k, \lambda_\gamma)] |1m'_s\rangle}{(n^0 - E_{d-p'} + i\epsilon) (p'^0 - E_{p'} + i\epsilon) (q'^2 - m_\pi^2 + i\epsilon)} V (d - p', p'; m'_s, m_d) ,
\end{aligned}$$

where $T_{\pi N}^1 (p_1, q; p', q')$ – pion-nucleon scattering amplitude acting as an operator on spin variables $|1m'_s\rangle$ and $|sm_s\rangle$ of a first nucleon in a two-nucleon system, $T_{\gamma N}^2 (p_2, q'; d - p', k, \lambda_\gamma)$ – π -meson photoproduction amplitude acting as an operator on spin variables $|1m'_s\rangle$ $|sm_s\rangle$ of a second nucleon in a two-nucleon system. n^0 , p'^0 , $E_{d-p'}$, $E_{p'}$ are “off-shell” and “on-shell” energy of nucleons with momenta $d - p'$, p' respectively. The term $[T_{\pi N}^1 T_{\gamma N}^2]$ is related to the sum over isospin variables in the two-particle operator:

$$[T_{\pi N}^1 T_{\gamma N}^2] = T_{\pi^- p \rightarrow \pi^- p}^1 T_{\gamma n \rightarrow p \pi^-}^2 - T_{\pi^0 n \rightarrow \pi^- p}^1 T_{\gamma p \rightarrow p \pi^0}^2 . \quad (6)$$

Due to isospin-0 of the deuteron the amplitudes of pion-nucleon scattering with and without the charge-exchange contribute to the re-scattering amplitude with opposite signs. The term $V(p'', p'; m'_s, m_d)$ is a Dnp -vertex function which in non-relativistic limit is connected to $\Psi_{m'_s, m_d}$ by [3]:

$$V(p'', p'; m'_s, m_d) = (E_D - E_{p'} - E_{p''}) \Psi_{m'_s, m_d} \left(\frac{1}{2} (p'' - p') \right), \quad (7)$$

where p' , p'' are momenta of nucleons in deuteron. In the expression (5) relativistic nucleon propagators were first expressed as a sum of the terms corresponding to virtual nucleons with positive and negative energy and then only the ones with positive energies were remained. This is because the influence of virtual nucleons with negative energy starts to manifest itself at momenta ~ 1 GeV [17], while such proton momenta in the kinematic region of Δ -isobar are not reached. One has to use relativistic form for the pion propagator in the kinematic region of Δ -isobar. The integrand has four poles of a variable p'^0 . Two poles are in the upper half-plane of complex variable p'^0 , the other two are in the lower half-plane:

$$\begin{aligned} p_{1+}^0 &= E_D - E_{d-p'} + i\epsilon, & p_{2+}^0 &= p_\Delta^0 - \omega_{q'} + i\epsilon, \\ p_{1-}^0 &= E_{p'} - i\epsilon, & p_{2-}^0 &= p_\Delta^0 + \omega_{q'} - i\epsilon, \end{aligned} \quad (8)$$

where p_Δ^0 is the energy of a πN -pair participating in the re-scattering, $\omega_{q'}$ - the “on shell” energy of a π -meson with a momentum q' . Integration over energy in (5) is done by closing up a contour in the lower half-plane, only a residue in a nucleon pole p_{1-}^0 is considered, while a residue in the pion propagator p_{2-}^0 can be neglected due to its smallness [3]. When integrating over 3-momentum of the nucleon the pion propagator is taking as:

$$\frac{1}{q'^2 - m_\pi^2 + i\epsilon} = P \frac{1}{q'^2 - m_\pi^2} - i\pi\delta(q'^2 - m_\pi^2), \quad (9)$$

and the expression (5) is expanded into a sum of terms corresponding to the contribution of δ - function and the main value of the integral:

$$\begin{aligned} T^{\pi N}(p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, m_d) = \\ T_{on}^{\pi N}(p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, m_d) + T_{off}^{\pi N}(p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, m_d), \end{aligned} \quad (10)$$

where :

$$\begin{aligned} T_{on}^{\pi N}(p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, m_d) = \\ = \frac{-1}{16\pi^2 |p_\Delta|} \int_0^{2\pi} d\phi' \int_{|p_-|}^{p_+} p' dp' \sum_{m'_s} \langle sm_s | \left[T_{\pi N}^1(p_1, q; p', q') T_{\gamma N}^2(p_2, q'; d - p', k, \lambda_\gamma) \right] | 1m'_s \rangle \times \\ \times \Psi_{m'_s, m_d} \left(\frac{1}{2} (d - 2p') \right) + \end{aligned} \quad (11)$$

$$+ \frac{(-)^{1+s}}{16\pi^2 |p_\Delta|} \int_0^{2\pi} d\phi' \int_{|p_-|}^{p_+} p' dp' \sum_{m'_s} \langle sm_s | [T_{\pi N}^1(p_2, q; p', q') T_{\gamma N}^2(p_1, q'; d - p', k, \lambda_\gamma)] | 1m'_s \rangle \times \\ \times \Psi_{m'_s, m_d} \left(\frac{1}{2} (d - 2p') \right),$$

$$T_{off}^{\pi N}(p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, m_d) = \tag{12} \\ -iP \int \frac{d^3 p'}{2\pi^3} \sum_{m'_s} \frac{\langle sm_s | [T_{\pi N}^1(p_1, q; p', q') T_{\gamma N}^2(p_2, q'; d - p', k, \lambda_\gamma)] | 1m'_s \rangle}{q'^2 - m_\pi^2} \Psi_{m'_s, m_d} \left(\frac{1}{2} (d - 2p') \right) + \\ + (-)^{1+s} iP \int \frac{d^3 p'}{2\pi^3} \sum_{m'_s} \frac{\langle sm_s | [T_{\pi N}^1(p_2, q; p', q') T_{\gamma N}^2(p_1, q'; d - p', k, \lambda_\gamma)] | 1m'_s \rangle}{q'^2 - m_\pi^2} \Psi_{m'_s, m_d} \left(\frac{1}{2} (d - 2p') \right) .$$

In the expression (11) the terms p_- and p_+ are given by:

$$p_\pm = \frac{|P_\Delta|}{Q} E_{c.m.} \pm \frac{P_\Delta^0}{Q} |p_{c.m.}|, \tag{13}$$

where Q is an invariant mass of πN pair, $E_{c.m.}$, $p_{c.m.}$ – nucleon energy and momentum in the πN center-of-mass frame, P_Δ^0 , P_Δ – energy and momentum of πN pair in a used frame. $|p_-|$ and p_+ are minimal and maximal nucleon momentum in a frame where energy and momentum of πN – pair are P_Δ^0, P_Δ . Note that the values of p_+ , p_- and kinematic variables in the expression (13) are different for the first and second terms of the expression (11).

In the amplitude (11) the integration over momentum p' is carried out in a frame where z -axis directed along the momentum of πN – pair, and the integration over $\cos(\theta')$ allows to get rid of δ – function and to fix the angle θ' .

The magnitude of the amplitude (11) strongly depends on an integration limit $|p_-|$. If a nucleon motion in the deuteron and the contribution of the deuteron D –state wave function are not included into consideration, then each term in the amplitude (11) in the Laboratory frame are proportional to the integral:

$$I_{lab.} = \int_{|p_-|}^{p_+} p' dp' u_0(p') . \tag{14}$$

In the kinematic region of Δ – isobar an upper limit of p_+ nearly everywhere exceeds 300 MeV/c, while a lower limit $|p_-|$ strongly depends on the momenta of nucleons and π –meson and varies in a range from 0 to 300 MeV/c. Therefore the integral (14) strongly increases at small $|p_-|$, while weakly depends on p_+ . In the center-of-mass frame of the reaction (1) an azimuthal dependence appears in an argument of the deuteron wave function, and each term in the amplitude (11) turns out to be proportional to the integral:

$$I_{c.m.} = \int_0^{2\pi} d\phi' \int_{|p_-|}^{p_+} p' dp' u_0 \left(\frac{1}{2} (|d - 2p'|) \right), \quad (15)$$

which also depends mainly on the lower integration limit $|p_-|$.

In the amplitudes $T_{on}^{\pi N}$ and $T_{off}^{\pi N}$ the residue in nucleon pole fixes one of nucleons on the mass shell. The second nucleon is not on the mass shell, however in the used kinematic region the shift from the mass shell is small. In the amplitude $T_{on}^{\pi N}$ the relation (9) fixes the π -meson on the mass shell. In the amplitude $T_{off}^{\pi N}$ the π -meson is not on the mass shell, but the major contribution to the integral comes from the region where the “on-shell” amplitude of π -meson photo-production and that of pion-nucleon scattering are applicable. The amplitudes (11), (12) were obtained by numerical integration. In the approximation used for the calculation of the main value of the integral the photo-production and pion-nucleon scattering amplitudes were factored out of integral for zero-momenta of nucleons inside the deuteron.

The diagram in Fig 1c and the same one but with identical nucleons permuted in final state correspond to the nucleon-nucleon re-scattering. The contributions of these two diagrams to the amplitude of the NN -re-scattering are equal in magnitude and have opposite signs. Since diagrams with permuted fermions contribute with opposite signs to the amplitude, these contributions are added and the expression for the NN -re-scattering amplitude in a coupled-basis approach is written as:

$$T^{NN}(p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, m_d) = \quad (16)$$

$$2 \int \frac{d^4 p'}{(2\pi)^4} \sum_{\substack{s'' m_s'' \\ m_s'}} \frac{\langle s m_s | T_{pp \rightarrow pp}^1(p_1, p_2; p', p'') | s'' m_s'' \rangle \langle s'' m_s'' | T_{\gamma n \rightarrow p\pi^-}^2(p'', q; d - p', k, \lambda_\gamma) | 1 m_s' \rangle}{(n^0 - E_{d-p'} + i\epsilon)(p'^0 - E_{p'} + i\epsilon)(p''^0 - E_{p''} + i\epsilon)} \times$$

$$\times V(d - p', p'; m_s', m_d),$$

where $T_{pp \rightarrow pp}^1(p_1, p_2; p', p'')$ is the proton-proton scattering amplitude, which acts as an operator on spin variables of both nucleons in two-nucleon systems $|s m_s\rangle$ and $|s'' m_s''\rangle$, $T_{\gamma n \rightarrow p\pi^-}^2(p'', q; d - p', k, \lambda_\gamma)$ is the π -meson photoproduction amplitude, acting as an operator on spin variables of a second nucleon in a two-nucleon system $|1 m_s'\rangle$ $|s'' m_s''\rangle$. The integration over p^0 is performed like it was done in case of πN - re-scattering by closing a loop in a lower half-plane and evaluating a residue in the nucleon pole $p^0 = E_{p'} - i\epsilon$. After the photoproduction and NN -scattering amplitudes are factored out of the integral and a contribution of D -state deuteron wave function is neglected the nucleon-nucleon re-scattering amplitude can be expressed as [3]:

$$\begin{aligned}
& T^{NN}(p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, m_d) = \\
& -i \frac{4\pi W}{m p_{c.m.}} \sum_{m_n m_{p'} m_{p''}} \langle \frac{1}{2} m_n \frac{1}{2} m_{p'} | 1 m_d \rangle \langle \frac{1}{2} m_{p''} \frac{1}{2} m_{p'} | s m_s \rangle T_{pp \rightarrow pp}(p_1, p_2, s, m_s; \frac{d}{2}, p'', s, m_s) \times \\
& \times T_{\gamma n \rightarrow p \pi^-}(p'', m_{p''}, q; \frac{d}{2}, m_n, k, \lambda_\gamma) \int \frac{d^3 \xi}{(2\pi)^3} \frac{(2\pi)^{\frac{3}{2}} \sqrt{2E_D} u_0 \left(\left| \xi + \frac{P}{2} \right| \right)}{\sqrt{4\pi}} \frac{(p_{c.m.}^2 + \beta^2)}{(p_{c.m.}^2 - \xi^2 + i\epsilon)(\xi^2 + \beta^2)} ,
\end{aligned} \tag{17}$$

where m_n , $m_{p'}$, $m_{p''}$ are z -projections of spins of intermediate state nucleons, W – energy of nucleon pair in center-of-mass frame, $p_{c.m.}$ – nucleon momentum in the same frame, $P = p_1 + p_2 - d$, $p'' = p_1 + p_2 - \frac{d}{2}$, $\beta = 241 \text{ MeV}$. The terms $T_{pp \rightarrow pp}(p_1, p_2, s, m_s; p', p'', s, m_s)$ are diagonal matrix elements of the pp -scattering amplitude in the channel-spin representation, they are expressed as a multipole expansion through phases of nucleon-nucleon scattering. The integral over ξ in (17) is evaluated analytically [3].

Double rescattering in the reaction (1) can be presented by diagrams in Fig. 2 and the similar diagrams obtained by permutating of identical nucleons in final state. We have calculated the contribution of these diagrams to the squared modulus of the amplitude and found that in the kinematics under study (Δ -resonance region, large momenta of final nucleons) it amounts about 1% of the contribution of the single rescattering.

Therefore we neglect double rescattering effects and the expression for the amplitude of the reaction (1) is written as:

$$\begin{aligned}
& T(p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, m_d) = T^{spect}(p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, m_d) + \\
& + T^{\pi N}(p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, m_d) + T^{NN}(p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, m_d) .
\end{aligned} \tag{18}$$

Usage of a π -meson on nucleon photoproduction amplitude evaluated in [12] provides a gauge invariance of the amplitude (18).

3

The amplitudes (18) are written in a mixed representation; polarization of particles in initial state is described by a helicity of γ -quantum λ_γ and z -projection of a deuteron spin m_d , while a final state polarization is described by a total spin of a nucleon pair s and its z -projection m_s . To calculate polarization observables it is more convenient to use helicity amplitudes [18]. A transition to helicity amplitudes is carried out using a unitary transformation from a coupled basis of the nucleon pair to its helicity basis and a transition from a projection of deuteron spin on z -axis, (which coincides with a direction of γ -quantum momentum) to the helicity of the deuteron:

$$\begin{aligned}
& T(p_1, \lambda_1, p_2, \lambda_2, q; k, \lambda_\gamma, d, \lambda_d) = \\
& = \sum_{sm_s} \sum_{m_1 m_2} \langle \frac{1}{2} m_1 \frac{1}{2} m_2 | s m_s \rangle D_{m_1 \lambda_1}^{*\frac{1}{2}}(\phi_1, \theta_1, 0) D_{m_2 \lambda_2}^{*\frac{1}{2}}(\phi_2, \theta_2, 0) (-)^{1-\lambda_d} T(p_1, p_2, q, s, m_s; k, \lambda_\gamma, d, -\lambda_d) ,
\end{aligned} \tag{19}$$

where $D_{m_i \lambda_i}^{\frac{1}{2}}(\phi_i, \theta_i, 0)$ are D -functions defined in [18], ϕ_i, θ_i – Euler angles, defining a transition to an individual helicity system of a nucleon i . The helicity amplitudes (19) are antisymmetric with respect to a permutation of final protons and they satisfy the relations following from the parity conservation:

$$T(p_1, \lambda_1, p_2, \lambda_2, q; k, \lambda_\gamma, d, \lambda_d) = \prod_i \eta_i (-)^{s_i - \lambda_i} T(p'_1, -\lambda_1, p'_2, -\lambda_2, q'; k', -\lambda_\gamma, d', -\lambda_d) , \quad (20)$$

where η_i , s_i are intrinsic parity and spin of a particle i , the momenta in a right-hand side of the equation are obtained from the initial ones by a reflection in xz -plane. In a case of coplanar kinematics these relations coincide with those for the two-particle reaction. The reaction (1) has in total 24 helicity amplitudes and to define them one has to know 47 independent observables (a common phase factor remains undetermined). For the coplanar kinematics the relations (20) decrease by a factor two the number of independent helicity amplitudes of the reaction (1) and their number becomes the same as for the two-particle reaction $\gamma d \rightarrow pn$.

A general expression for polarization observables of the three-particle reaction (1) is defined according to [18]:

$$F_{I_\gamma M_\gamma, I_d M_d}^{I_1 M_1, I_2 M_2} = \frac{Sp T \tau_{I_\gamma M_\gamma} \tau_{I_d M_d} T^\dagger \tau_{I_1 M_1} \tau_{I_2 M_2}}{Sp T T^\dagger} , \quad (21)$$

where T – helicity amplitudes (19), $\tau_{I_i M_i}$ – spherical spin-tensor of a nucleon having a momentum p_i , $\tau_{I_\gamma M_\gamma}, \tau_{I_d M_d}$ – spherical spin-tensors of γ -quantum and deuteron respectively. One has to note that for a given energy of the initial state polarization observables of a two-particle reaction are functions of the single kinematic variable – scattering angle, while for a three-particle reaction they are functions of five variables and their choice is ambiguous. Often for such variables one takes momentum and two escape angles of one particle and two escape angles of other particle.

4

To demonstrate an influence of re-scattering effects we present the results of calculation of the squared moduli of amplitudes corresponding to the diagrams in Fig. 1, analyzing powers of the reaction (1) connected to beam polarization $T_{22,00}$, to target polarization $T_{00,20}$ and to polarization of one of the final protons P_{1y} . The calculations were performed in a center-of-mass frame of the reaction (1) for the coplanar kinematics. These parameters are shown as functions of proton escape angle θ_2 for fixed values of the following five variables: $p_1 = 300 \text{ MeV}$, $\theta_1 = 100^\circ$, $\phi_1 = 0^\circ$, $\phi_2 = 180^\circ$ and γ – quantum energy $E_\gamma = 360 \text{ MeV}$.

Fig. 3 it is shown a dependence of squared moduli of the amplitudes averaged over the spins of initial particles and summed over the spins of final particles. For the proton escape angle θ_2 larger than 120° the contribution of the re-scattering is small if compared to that of the spectator mechanism, therefore the dependence is shown only in the θ_2 range between 0° and 120° . The squared modulus of the “on-shell” πN – re-scattering amplitude shows a most characteristic behavior. It has a clear maximum in the region $\theta_2 \sim 40^\circ$. This maximum comes from the first term of the amplitude (11) corresponding to the case when a proton with a momentum p_1 takes part in the πN – re-scattering. In this kinematic region the value of $|p_-|$ in the first term of (11) is small and the integral (15) reaches the maximum. The main contribution to the “off-shell” πN – re-scattering comes from the first term of the amplitude (12) which, like in the previous case, corresponds to the re-scattering of a proton with the momentum p_1 . The behavior of the “off shell” πN – re-scattering amplitude is defined mainly by a dependence of the integral

$$I_{off} = P \int d^3p' \frac{u_0 \left(\frac{1}{2} (|d - 2p'|) \right)}{q'^2 - m_\pi^2}$$

on the proton escape angle θ_2 . The contribution of the nucleon-nucleon re-scattering increases drastically with decreasing a relative kinetic energy of final nucleons because a scattering phase in 1S_0 state grows. In Fig.3 this corresponds to the increase of the contribution of the nucleon-nucleon re-scattering in a region of small θ_2 . A contribution of the 1S_0 –state re-scattering decreases with increasing the relative kinetic energy, but in the same time a contribution of the scattering in the states with orbital momenta $L = 1, 2$: $^3P_0, ^3P_1, ^3P_2, ^1D_2$ grows.

In Fig.2 an increase of a contribution of the nucleon-nucleon re-scattering in a region $\theta_2 \sim 90^\circ$ is determined by a re-scattering in P – and D – states.

In Figs. 3,4 is shown an influence of the re-scattering on the tensor analyzing power connected to the target polarization, $T_{00,20} = F_{00,20}^{00,00}$, and the one connected to the beam polarization, $T_{22,00} = F_{22,00}^{00,00}$. The major contribution comes from the “on shell” πN – re-scattering which leads to a deep minimum in the region $\theta_2 \sim 40^\circ$. When the “off shell” πN – re-scattering and NN – re-scattering are taken into account the depth of the minimum decreases, this is especially true for $T_{22,00}$. Note that a decrease of the depth of the minimum in $T_{22,00}$ is connected with the “off shell” πN – re-scattering, although it is in the region $\theta_2 \sim 40^\circ$ that squared modulus of its amplitude has a minimum. The major contribution to the decrease of the depth of the minimum of $T_{22,00}$ comes from an interference between “of-shell” and “on-shell” πN –re-scattering amplitudes.

In Fig.5 one can see an influence of the re-scattering on the polarization of a final proton having a momentum p_1 for an unpolarized initial state, which is defined as:

$$P1_y = \frac{SpTT^\dagger \sigma_y(1)}{SpTT^\dagger} ,$$

where T are helicity amplitudes (19), $\frac{1}{2}\sigma_y(1)$ – operator of a y -projection of the proton spin, other components of the polarization vector in coplanar kinematics are equal to zero. One can see that the re-scattering effects noticeably changes the behavior of $P1_y$, especially in a region $\theta_2 \sim 40^\circ - 60^\circ$. Note that in the reaction (1) with unpolarized initial state the proton polarization is non-zero even in the spectator model without accounting for $\pi N - NN$ – re-scattering.

This can be explained by a necessity to include besides Born terms a contribution of Δ – isobar in s - and u - channel as well as ρ -meson and ω -meson exchange in t -channel for the description of the photoproduction of π -meson on nucleon. An account of the contribution of Δ – isobar in s – channel leads to the appearance of an imaginary part in the amplitude of the photoproduction of π -meson on nucleon, and as a consequence, to the polarization of a final nucleon with an unpolarized initial state. In the same time for the reaction $e^-d \rightarrow e'^-pn$ the polarization of final nucleons with an unpolarized initial state turns out to be non-zero only if one takes into consideration pn – re-scattering [?], because the eN -re-scattering amplitude is real in Born approximation.

Calculation of the influence of pion-nucleon and nucleon-nucleon re-scattering on polarization observables of the reaction $\gamma d \rightarrow pp\pi^-$, performed in the framework of diagrammatic approach, has shown that the effects of re-scattering play a noticeable role in the behavior of the polarization observables in the kinematic region of Δ -isobar with large momenta of protons in the final state. Contribution of the re-scattering effects must be taken into account in the analysis of experimental data.

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$$|T_{fi}|^2, 10^{-8} MeV^{-4}$$

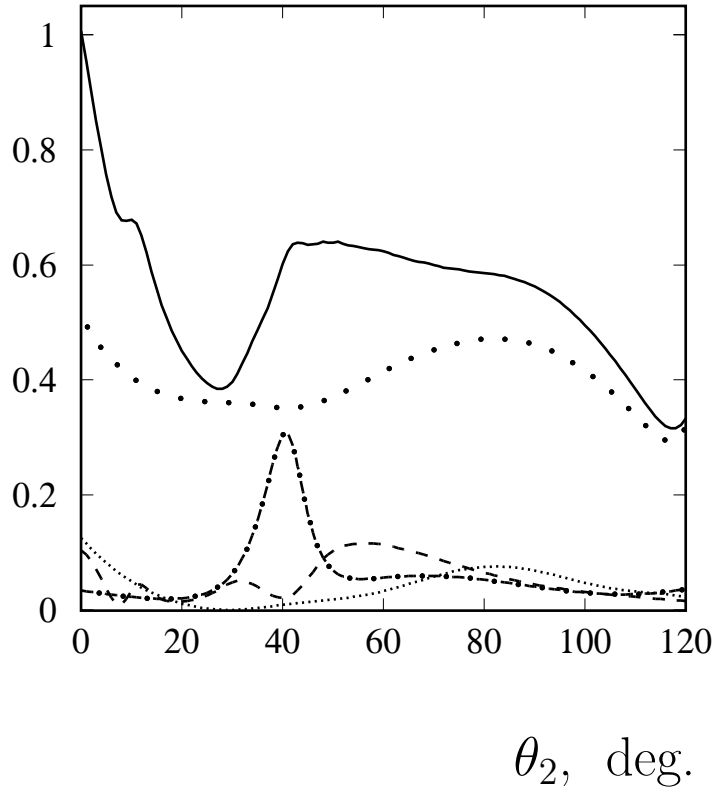


Figure 2: Dependence of squared moduli of the amplitudes on the proton escape angle θ_2 for $p_1 = 300 MeV/c$, $\theta_1 = 100^\circ$, $\phi_1 = 0^\circ$, $\phi_2 = 180^\circ$, $E_\gamma = 360 MeV$ in the c.m. of the reaction. The large dotted line is spectator model calculation, dash-dotted line is on-shell πN re-scattering, dashed line is off-shell πN re-scattering, small dotted line is NN re-scattering, solid line includes all diagrams of Fig.1

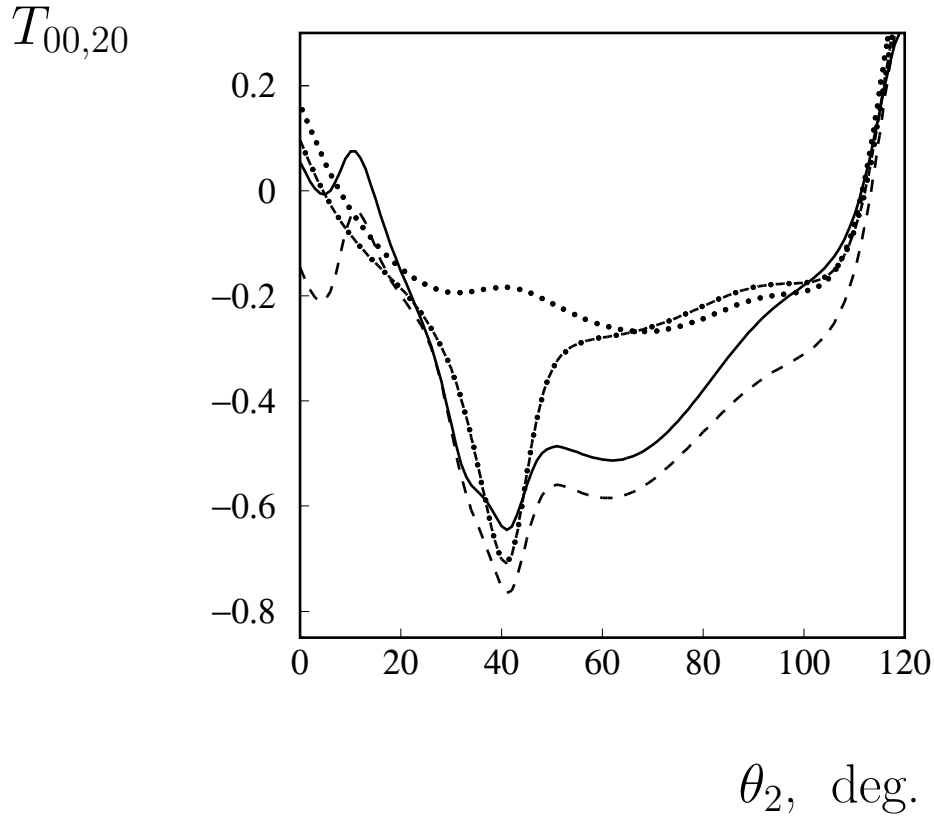


Figure 3: Influence of the re-scattering effects on the tensor analyzing power connected to the target polarization. Kinematics is the same as in Fig.2. The dotted line is spectator model calculation, dash-dotted line includes on-shell πN re-scattering, dashed line includes on-shell and off-shell πN re-scattering, solid line includes on-shell and off-shell πN re-scattering and NN re-scattering.

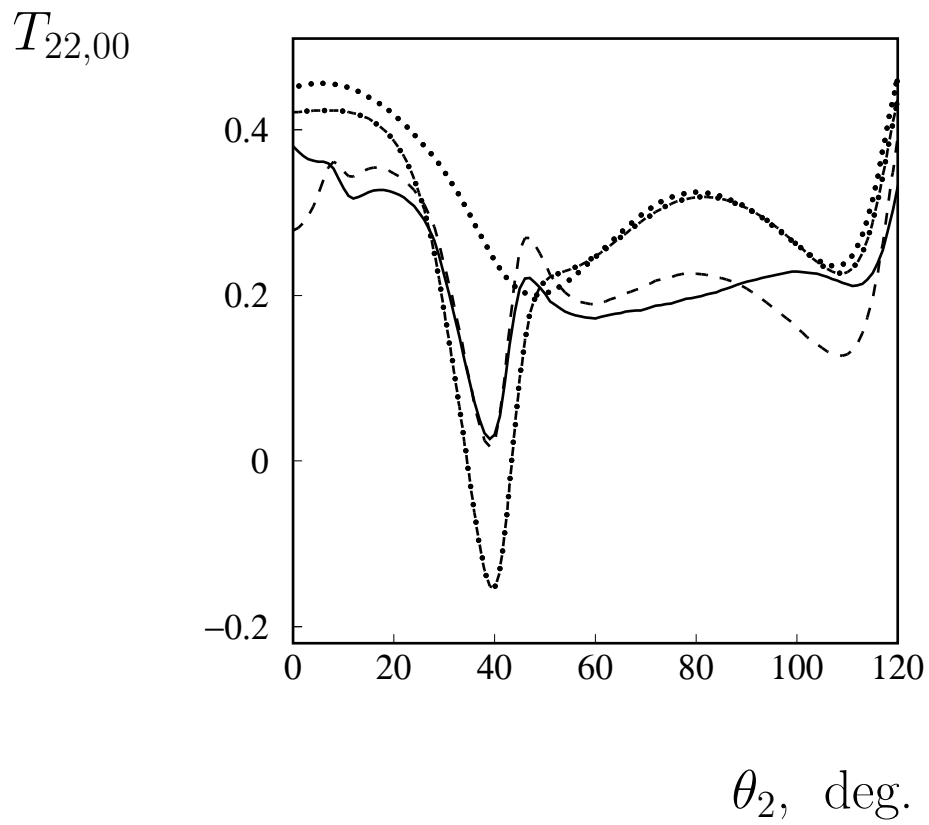


Figure 4: Influence of the re-scattering effects on the tensor analyzing power connected to the beam polarization. Kinematics is the same as in Fig.2, the curve definitions are the same as in Fig.3.

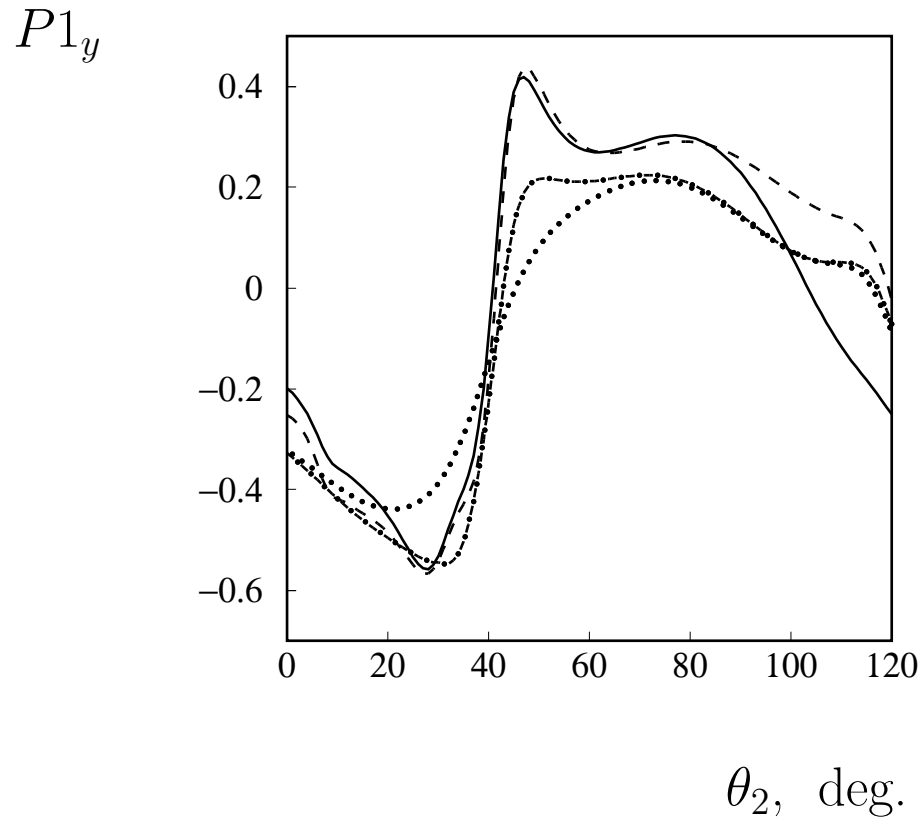


Figure 5: Influence of the re-scattering effects on the nucleon polarization. Kinematics and the curve definitions are the same as in Fig.4.